Random multi-player games in complex networks

Keywords: Evolutionary games, complex networks.

Authors: Natalia Kontorovsky, Juan Pablo Pinasco, Federico Vázquez.

Evolutionary game theory provides a framework to study the behavior of large populations where individuals playing different strategies (or having different biological traits) interact through some game, and they can replicate according to their payoffs.

Evolutionary games with pairwise interactions were extensively studied [1, 2]. Local interactions with various opponents had also been considered, where in each round the same fixed number of players are randomly selected from the population to play against each other. However, there are many situations in which the number of players can vary over time and even between rounds. It also happens that the optimal strategies in a two players game could not be optimal in a three players game, thus interactions between multiple players can not be reduced to pairwise interactions. Therefore, it is interesting to model and study the case in which the game can be played by a different number of players in each round when the strategy must be selected previously, without knowing a priori the exact number of players involved.

The concept of evolutionary stable strategy (ESS) is a central when studying time evolution, because it satisfies the additional stronger condition of stability, which implies that if an ESS is reached, then the proportions of players playing the different strategies do not change over time

In this work [3] we formalize and generalize the definition of evolutionary stable strategy (ESS) to be able to include a scenario in which the game can be played by a different number of players in each round. Even though a similar problem was analyzed previously in terms of two types of players, incumbents (original population) and mutants (invaders)[4], only two combinations of them were considered. Here we show that all combinations must be considered, and a hierarchy of payoffs is needed in order to characterize an ESS when the number of players in each interaction is a random variable.

In order to explore these questions, we study the simplest non-trivial case of the duel-truel game. As usual, in a duel two players aim to eliminate each other, while in a truel three players are involved. Each player can use one of two possible strategies, that we call *perfect and mediocre* strategies. When a player uses the strategy perfect, then it annihilates its competitors with probability 1.0, while a player using strategy mediocre kill its opponents with probability 0.5. The paradox is that, when a truel game is played, a perfect player is not necessarily the winner of the game, even having the highest killing probability.

This surprising result was already present in the early literature on truels [5]. As a consequence, in a duel game the ESS corresponds to the entire population playing strategy perfect, whereas in a truel game the ESS corresponds to all players using strategy mediocre.

This led us to consider what the ESS would be in a scenario where the number of players is a random variable, the so-called Poisson games, where at each iteration step of the dynamics a duel is played with probability $p \in (0, 1)$, and a truel is played with the complementary probability $1 - p$.

We also introduce an agent-based model in which players interact in a complex network by copying the strategies of their neighbors, with a dynamics that in mean-field evolves following the replicator equation. Let us observe that the replicator dynamics is usually defined in terms of new individuals entering the population, by selecting a pure strategy with a probability proportional to the payoff given the current mix of agents. Our approach has an independent interest, and has the advantage that can be used in networks with a fixed number of nodes or agents, bypassing the issue of how to add new nodes as agents replicate.

We perform extensive Monte Carlo (MC) simulations of the model in different types of networks, and develop an analytical approach based on a pair approximation (PA) that allows to obtain approximate equations for the evolution of the fraction of perfect agents in the network. This approach enable us to investigate if the transitions between ESS in pure and mixed strategies found within the Nash equilibrium theory are also observed in complex networks, and to identify how the networks' topology affects the existence of mixed equilibria.

In Fig. 1 we can see that the coexistence of perfect and mediocre players predicted by the mean-field approach when interactions are all-to-all (dashed line) is also present when agents interact in complex topologies (solid lines), and have a good agreement with MC simulations (symbols). Figure 2 shows the phase diagram in the p–μ space, where μ is the mean degree of the network. We observe that the coexistence phase shrinks as μ decreases, but it does not seem to vanish completely even for small values of μ. As a consequence, a given unstable mix of the two types of players for some value of p, can turn into stable when the mean number of neighbors of a player is increased beyond a threshold. This result implies that the network of interactions affects the stability of the system by inducing a stable coexistence when its connectivity increases.

References

[1] J. P. Pinasco, M. Rodriguez Cartabia, and N. Saintier, "Evolutionary game theory in mixed strategies: From microscopic interactions to kinetic equations," Kin. Relat. Models 14, 115–148 (2021).

[2] J. M. Smith, "Evolutionary game theory," Physica D 22, 43–49 (1986).

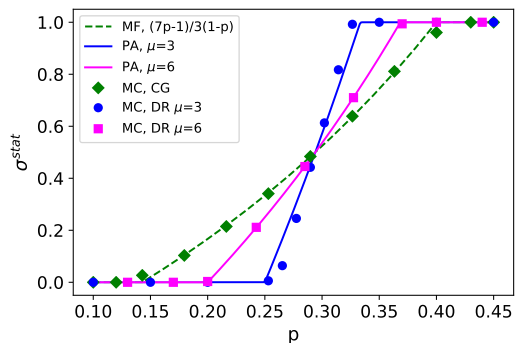

Figure 1: Stationary fraction of perfect agents σ^{stat} vs duel probability p, for the values of the mean degree μ indicated in the legend. The dashed line corresponds to the stable solution σ_{CG} on a Complex Graph (CG), while solid lines represent the solution from the PA equations. Symbols correspond to the average value of σ at the stationary state obtained from MC simulations on a CG of size $N = 10^3$ (diamonds), and DRRGs of size $N = 10^4$ and degrees $\mu = 6$ (squares) and $\mu = 3$ (circles).

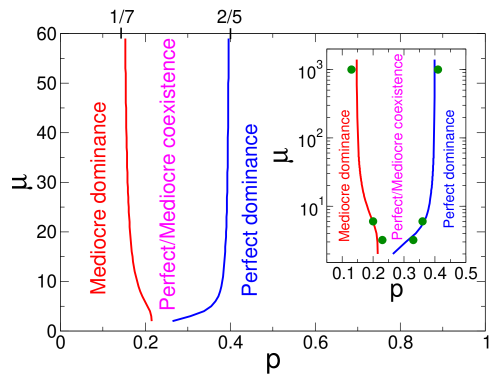

Figure 2: Phase diagram on the p–μ space showing the transition lines between the coexistence and dominance phases, obtained from the PA equations for an ER network.

[3] Kontorovsky, N. L., Pinasco, J. P., Vazquez, F. (2022). Random multi-player games. Chaos: An Interdisciplinary Journal of Nonlinear Science, 32(3), 033128.

[4] H. Tembine, E. Altman, R. El-Azouzi, and Y. Hayel, "Evolutionary games with random number of interacting players applied to access control," in 2008 6th International Symposium on Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks and Workshops (IEEE, 2008) pp. 344– 351.

[5] P. Amengual and R. Toral, "Truels, or survival of the weakest," Computing in Science Engineering 8, 88–95 (2006).